Inferring Private Valuations from Behavioral Data in Bilateral Sequential Bargaining

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Background

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- E-commerce platforms: eBay, Xianyu.

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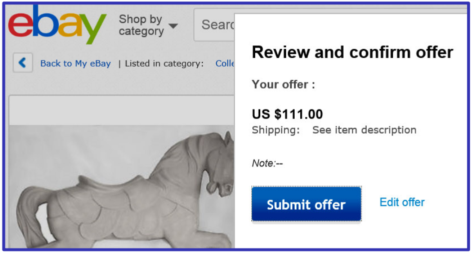

There are over 90 million such listings on eBay during 2012∼2013.

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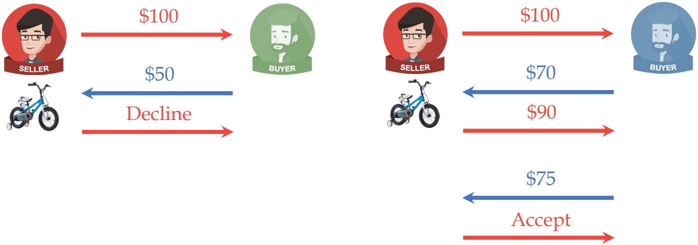

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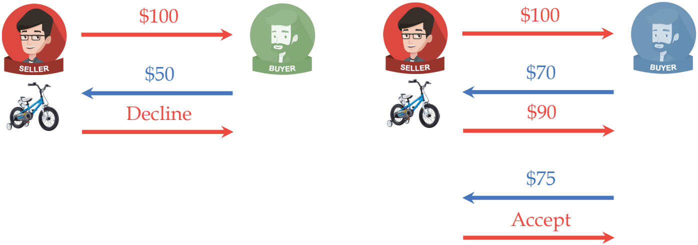

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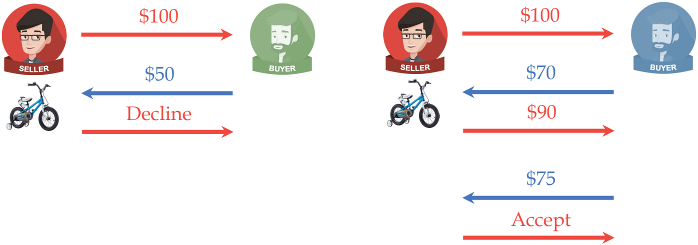

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Problem Description

- We first focus on one seller who sells multiple items.
- Denote observable data as $\{(\boldsymbol{x}_i^m, y_i^m)\}_{i\in\mathcal{I},m\in\mathcal{M}}$
  - $\boldsymbol{x}_i^m$: history of the bargaining (between the seller and a buyer)
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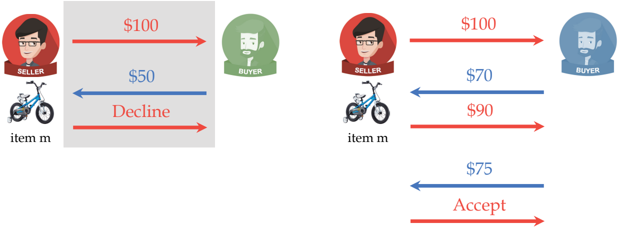

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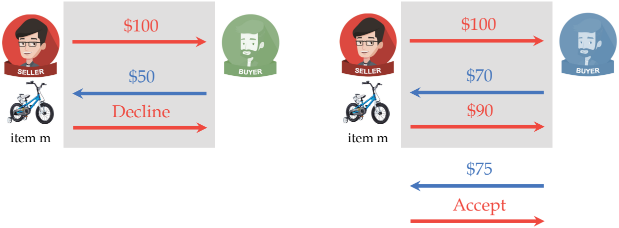

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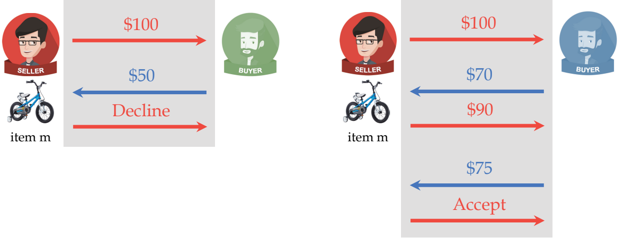

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- Denote seller's valuation for item m as v^m (unobservable).

Private Valuation Inference Problem

Given $\{(\boldsymbol{x}_i^m, y_i^m)\}_{i\in\mathcal{I}, m\in\mathcal{M}}$, how to infer $\{v^m\}_{m\in\mathcal{M}}$?

Solution

Seller Behavior Function

- Use $f_{\theta}(\cdot,\cdot)$ to denote seller's behavior function

$$\underbrace{f_{\theta}\left(v^m, \mathbf{x}_i^m\right)}_{\text{distribution of predicted decision}} \rightarrow \underbrace{y_i^m}_{\text{observed decision}}$$

- If θ are known, we can infer v^m using Bayes' Rule:

$$\begin{aligned}&\Pr\left(v^m = v \middle| \left\{\left(\mathbf{x}_i^m, y_i^m\right)\right\}_{i\in\mathcal{I}}, \theta\right)\\ &= \frac{\Pr_{\text{prior}}\left(v^m = v\right)\Pr\left(\left\{y_i^m\right\}_{i\in\mathcal{I}} \middle| v^m = v, \left\{\mathbf{x}_i^m\right\}_{i\in\mathcal{I}}, \theta\right)}{\sum_{\tilde{v}}\Pr_{\text{prior}}\left(v^m = \tilde{v}\right)\Pr\left(\left\{y_i^m\right\}_{i\in\mathcal{I}} \middle| v^m = \tilde{v}, \left\{\mathbf{x}_i^m\right\}_{i\in\mathcal{I}}, \theta\right)}.\end{aligned}$$

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- Our solution: Model $f_{\boldsymbol{\theta}}(\cdot,\cdot)$ via GRU ($\boldsymbol{\theta}$ are trainable weights).
  - Challenge: How to train GRU on $\left\{\left(\mathbf{x}_i^m, y_i^m\right)\right\}_{i,m}$ to get θ?
    It is not a standard supervised learning problem.

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- Assumption: seller never chooses strictly dominated decisions.
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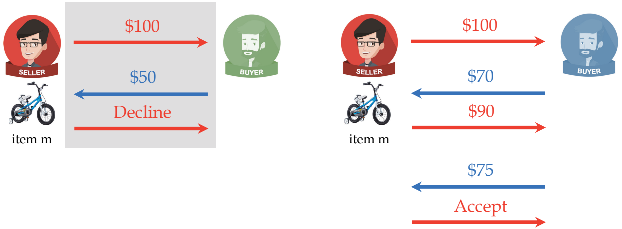

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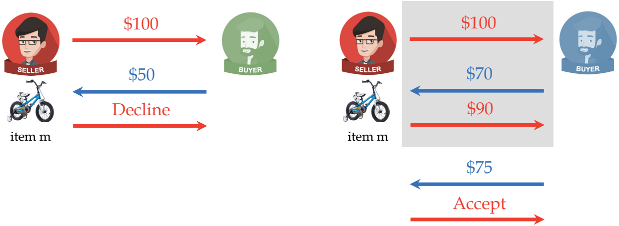

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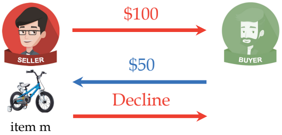

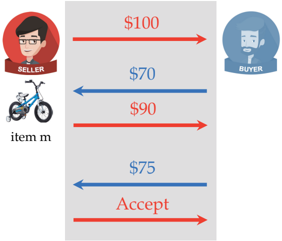

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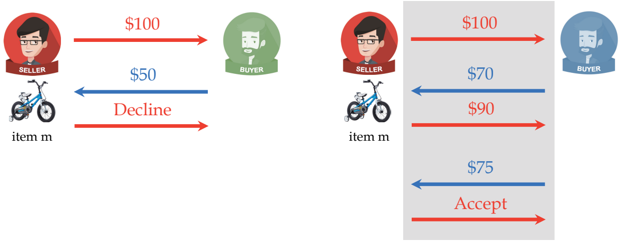

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Learning of θ

- We learn $\boldsymbol{\theta}$ by minimizing following loss function on data:

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Extension: Heterogeneous f_{θ_k}

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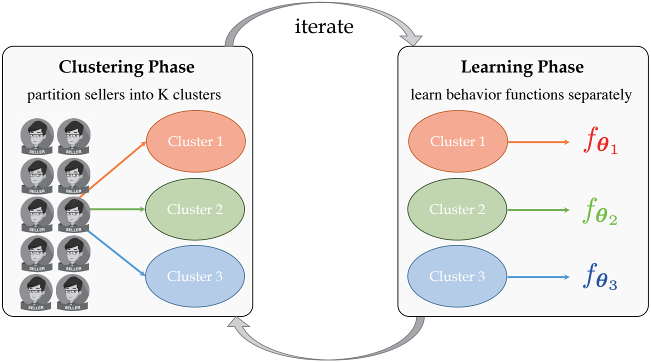

Experiments

Experimental Settings

- Comparison methods
  - Our method (homo./hetero.)
  - Single learning (homo./hetero.)
  - Dual learning (homo./hetero.)
- Datasets
  - Synthetic dataset: 900 sellers, 120,000 bargaining threads
    - We use different theoretical models to simulate human behaviors.
    - Ground truth (i.e., v^m) is known.
  - Real dataset: 30,000+ sellers, 300,000+ bargaining threads
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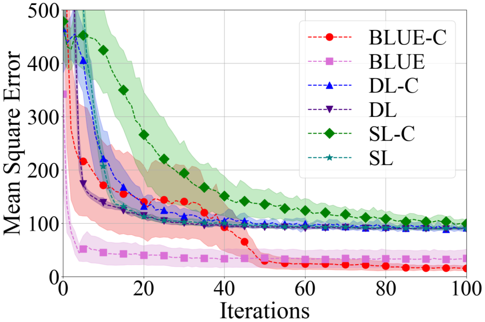
Results

- Synthetic validation data (10% of all synthetic data)
  - The randomly generated v^m belongs to $\{10, 14, \ldots, 94, 98\}$.



MSEs of Inferred v^m Under Different Schemes (Averaged Over Six Runs).

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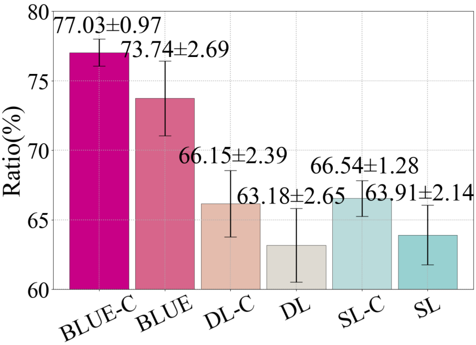
  - Ratio: fraction of inferred v^m satisfying secrete bounds.



Ratios Under Different Schemes (Averaged Over Six Runs).

Conclusion

- We propose a private valuation inference method based on ML.
  - Challenge: valuation is not observable.
  - Novelty: define feasible interval, and include it in a new loss function to guide learning.

Publication

- Lvye Cui and Haoran Yu, "Inferring Private Valuations from Behavioral Data in Bilateral Sequential Bargaining," International Joint Conference on Artificial Intelligence (IJCAI), Macao, China, August 2023.

THANK YOU