

# Analyzing Location-Based Advertising for Vehicle Service Providers Using Effective Resistances

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# Background

- Vehicle service providers have recently shown ads in vehicles.
- [This talk](#): Understand how in-vehicle ads impacts service prices.

# In-Vehicle Advertising: Example

**Example 1 (Taxis):** Curb Inc. installs tablets in taxis in 10 major cities in the U.S.



- Passengers can watch **TV programs**.
- Taxi companies can generate extra revenue by displaying **ads**.

# In-Vehicle Advertising

**Example 2 (Ride-Sharing Systems):** VUGO Inc. installs tablets for Uber & Lyft drivers, displays ads based on origins & destinations, and shares ad revenue with drivers.



# In-Vehicle Advertising

**Example 3 (Bike-Sharing Systems):** Mobike Inc. recently tested location-based advertising in Shanghai.



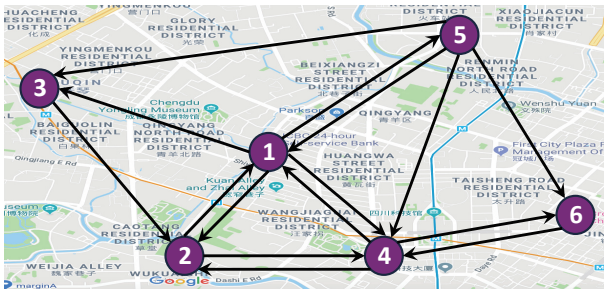
# Problem

- First, we describe basic settings (e.g., traffic graph and prices).
- Second, we raise one key question about in-vehicle advertising.



# Problem Description (Traffic Graph and Prices)

- We focus on a vehicle service provider who **owns** vehicles.
- Traffic graph (**node**: location, **link**: traffic demand)

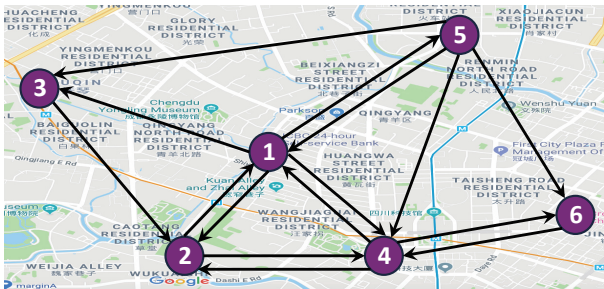


- Provider sets different vehicle service prices for different links. Let  $p_{ij}$  be the price for link  $(i, j)$  ( $i$ : origin;  $j$ : destination).
  - e.g.,  $p_{13} = \$1/\text{minute}$ .
    - Can be converted to  $\$/\text{mile}$  based on vehicle velocity.



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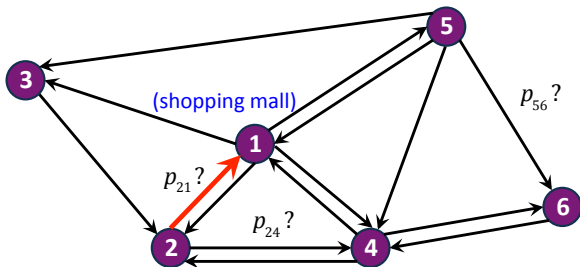


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## Q: How Does Advertising Impact Prices in Network?

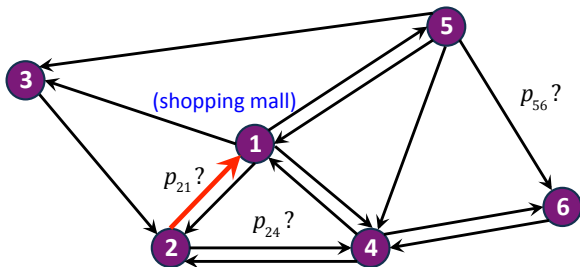
- **Example:** Provider collaborates with an advertiser (**shopping mall**) and gets ad revenues by displaying ads to riders on (2, 1).



- How should the provider change its service prices?
  - **Reduce**  $p_{21}$  to increase the number of riders on (2, 1)?
  - **Increase**  $p_{24}$  to save vehicles on (2, 4)?
  - How about  $p_{56}$ ? Non-negligible impact?
- **This talk:** (i) Derive expressions of prices (e.g.,  $p_{21}$ ,  $p_{24}$ ,  $p_{56}$ );  
(ii) Analyze advertising's impact on prices.

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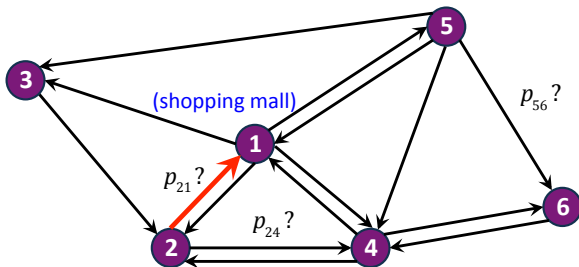
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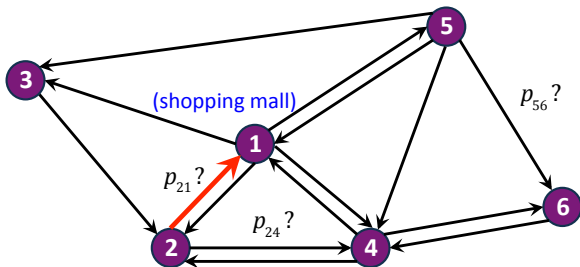
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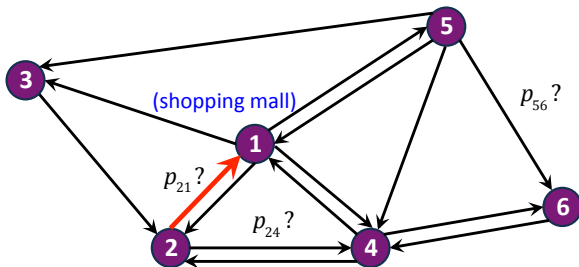
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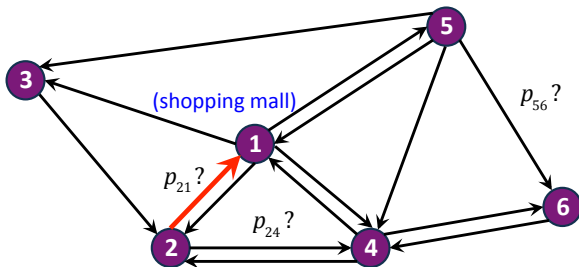
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# Related Work

- Some prior work on advertising's impact on service prices:
  - **Media service:** [Kaiser and Wright 2006], [Peitz and Valletti 2008], [Godes *et al.* 2009], [Anderson and Jullien 2015]
  - **Location-based service:** [Yu *et al.* 2017]
  - **Mobile app service:** [Guo *et al.* 2018]
- Our work focuses on **vehicle service**.
  - There are **multiple prices** (each link in network has a price).
  - The advertising's impact is affected by the **network topology**.

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# Model

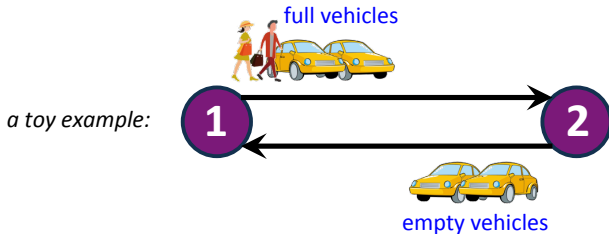
Formulate provider's pricing problem considering ad revenues.

# Model (Notations)

- **Traffic network's parameters** (constants)
  - $\theta_{ij} \geq 0$ : number of users **considering** taking vehicle service on  $(i, j)$  **in each time slot** (e.g., one time slot = one minute).
  - $\xi_{ij} > 0$ : travel time from  $i$  to  $j$  (measured by number of slots).
- **Provider's decision variables**
  - $p_{ij}$ : service price for  $(i, j)$  (\$ per time slot).
  - Routing **full vehicles** (carrying users), **empty vehicles** (no users)
    - $q_{ij}^{\text{full}} \geq 0$ : **full vehicles'** departure rate for  $(i, j)$ .
    - $q_{ij}^{\text{empty}} \geq 0$ : **empty vehicles'** departure rate for  $(i, j)$ .

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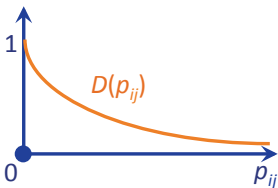


# Model (Constraints)

- Demand constraint (decision variables are in blue)

$$q_{ij}^{\text{full}} \leq \theta_{ij} D(p_{ij}), \forall i, j.$$

- $D(p_{ij})$ : fraction of users accepting  $p_{ij}$  and taking vehicle service.



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Parameter:  $\theta_{ij}$ : number of users considering service per slot.  
Decisions:  $q_{ij}^{\text{full}}$ : full vehicle routing;  $p_{ij}$ : vehicle service price.

# Model (Constraints)

- Vehicle flow balance constraint

$$\underbrace{\sum_j \left( q_{ij}^{\text{full}} + q_{ij}^{\text{empty}} \right)}_{\text{rate of vehicles departing from } i} = \underbrace{\sum_j \left( q_{ji}^{\text{full}} + q_{ji}^{\text{empty}} \right)}_{\text{rate of vehicles arriving at } i}, \forall i.$$

This couples the provider's decisions for different links.

---

Decisions:  $q_{ij}^{\text{full}}, q_{ij}^{\text{empty}}$ : full/empty vehicle routing.

# Model (Objective)

- **Objective** (time-average profit from all links)

$$\max \sum_{(i,j)} \left( \underbrace{\xi_{ij} q_{ij}^{\text{full}}}_{\text{number of full vehicles running on } (i,j) \text{ in any slot}} \left( p_{ij} + a_{ij} - c^{\text{full}} \right) - \xi_{ij} q_{ij}^{\text{empty}} c^{\text{empty}} \right)$$

- Some new parameters:
  - $a_{ij} \geq 0$ : ad revenue **per full vehicle per time slot** on  $(i, j)$ .
  - $c^{\text{full}}, c^{\text{empty}} > 0$ : a (full/empty) vehicle's operation cost **per slot**.

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Parameter:  $\xi_{ij}$ : travel time.

Decisions:  $q_{ij}^{\text{full}}, q_{ij}^{\text{empty}}$ : full/empty vehicle routing;  $p_{ij}$ : vehicle service price.



# Problem Formulation

- The provider's problem:

$$\max \sum_{\text{each link } (i,j)} \left( \xi_{ij} q_{ij}^{\text{full}} (p_{ij} + a_{ij} - c^{\text{full}}) - \xi_{ij} q_{ij}^{\text{empty}} c^{\text{empty}} \right)$$

$$\text{s.t. } q_{ij}^{\text{full}} \leq \theta_{ij} D(p_{ij}), \forall i, j, \text{ (demand constraint)}$$

$$\sum_j (q_{ij}^{\text{full}} + q_{ij}^{\text{empty}}) = \sum_j (q_{ji}^{\text{full}} + q_{ji}^{\text{empty}}), \forall i, \text{ (flow balance)}$$

$$\text{var. } q_{ij}^{\text{full}}, q_{ij}^{\text{empty}} \geq 0, p_{ij}, \forall i, j.$$

- Question: What is  $\frac{\partial p_{ij}^*}{\partial a_{xy}}$  ( $p_{ij}^*$  is optimal price;  $a_{xy}$  is ad revenue)?
  - Hard to directly compute  $p_{ij}^*$ : non-convex problem in general.
- Rest of talk
  - [Solution] design  $p_{ij}^\phi$  and analyze  $\frac{\partial p_{ij}^\phi}{\partial a_{xy}}$ ;
  - [Performance] study  $p_{ij}^\phi$ 's optimality theoretically & numerically.
    - $p_{ij}^\phi$  achieves close-to-optimal profit  $\implies \frac{\partial p_{ij}^\phi}{\partial a_{xy}}$  gives insights.

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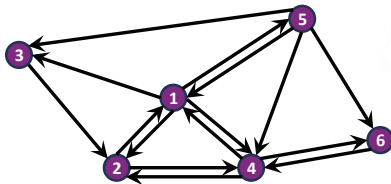
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## Solution

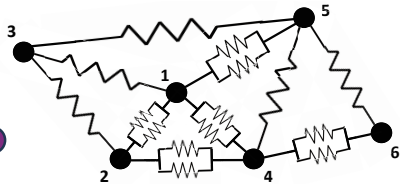
- We propose an innovative design of  $p_{ij}^\phi$ .
  - **Vehicle networks** are similar to **electrical networks** (e.g., keeping vehicle/current flow balance at each node).
  - We will borrow notions from **electrical networks** to design  $p_{ij}^\phi$ .

# Construction of Electrical Network

- Construct an **electrical network** based on traffic network.



Traffic Network



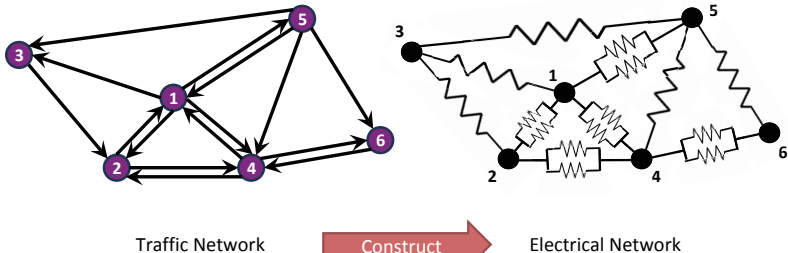
Electrical Network

- link  $(i, j) \implies$  a resistor with resistance  $r_{ij} = \frac{\xi_{ij}}{\theta_{ij}}$ .

Parameters:  $\theta_{ij}$ : number of users considering service per slot;  $\xi_{ij}$ : travel time.

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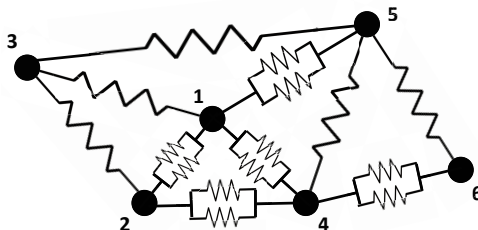


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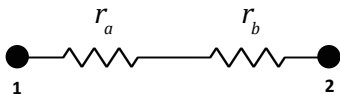
# Effective Resistances

- Given an electrical network described by  $\{r_{ij}\}_{i,j}$ , we can compute *Effective Resistance* between any two nodes  $i$  and  $j$ .



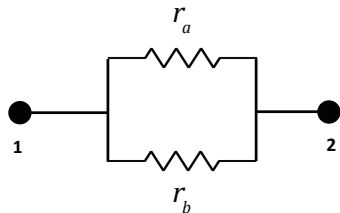
# Effective Resistances

- Examples of computing *Effective Resistance*.



effective resistance  
between 1 and 2

$$R_{12} = r_a + r_b$$

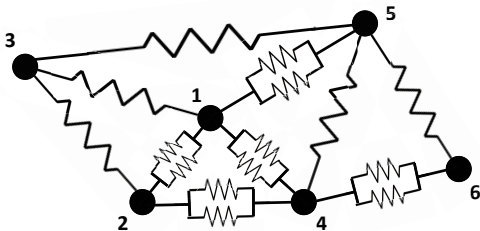


$$R_{12} = \frac{r_a r_b}{r_a + r_b}$$



# Effective Resistances

- Let  $R_{ij}(\theta, \xi)$  denote the **effective resistance** between  $i$  and  $j$  (can be computed in polynomial time).
  - e.g.,  $R_{34}(\theta, \xi)$ 's value depends on all resistors.



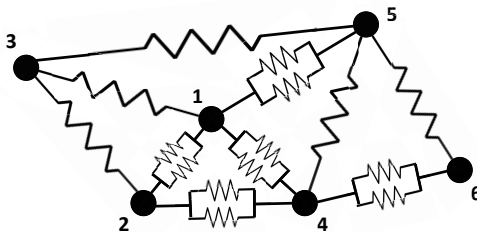
- $R_{ij}(\theta, \xi)$  internalizes the network topology's influence.
  - Intuition: small  $R_{ij}(\theta, \xi) \iff$  easy to route vehicles from  $i$  to  $j$ .
- We design prices based on  $\{R_{ij}(\theta, \xi)\}_{i,j}$ .

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# Our Resistance-Based Pricing

## Resistance-Based Pricing

Let  $\phi > 0$  be a control parameter. Our resistance-based price for  $(i, j)$  is given by

$$p_{ij}^{\phi} = \frac{1}{2} \underbrace{\left( \frac{1}{\phi} - a_{ij} + c^{\text{full}} \right)}_{\text{parameters of link } (i,j)} + \frac{1}{4\xi_{ij}} \sum_x \sum_y \underbrace{\rho_{ijxy}(\theta, \xi)}_{\text{weight function}} \underbrace{\theta_{xy} \left( \frac{1}{\phi} + a_{xy} - c^{\text{full}} \right)}_{\text{parameters of link } (x,y)},$$

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# Advertising's Impact on Prices

- If  $(x, y) \neq (i, j)$ , we have

$$\frac{\partial p_{ij}^{\phi}}{\partial a_{xy}} = \frac{\theta_{xy}}{4\xi_{ij}} (R_{jx}(\theta, \xi) - R_{ix}(\theta, \xi) - R_{jy}(\theta, \xi) + R_{iy}(\theta, \xi)).$$

- Come back to our example:

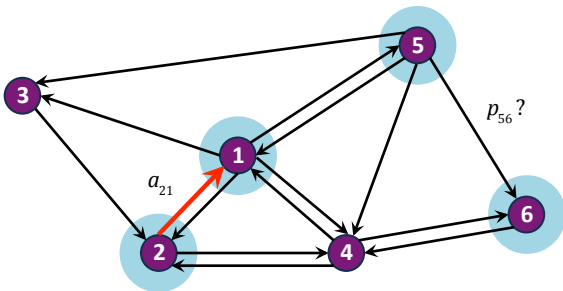
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Sign of  $\frac{\partial p_{56}^{\phi}}{\partial a_{21}}$  depends on  $R_{62}(\theta, \xi) - R_{52}(\theta, \xi) - R_{61}(\theta, \xi) + R_{51}(\theta, \xi)$ .

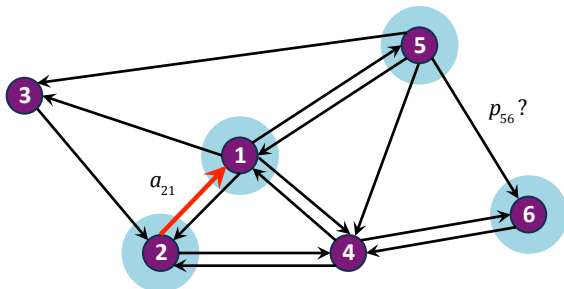
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# Performance

- (Theoretical) If demand function  $D$  is *linear* and provider *cannot route empty vehicles*,  $p_{ij}^\phi$  is *optimal*.
- (Experimental) If demand function  $D$  is *exponential* and provider *can route empty vehicles*,  $p_{ij}^\phi$  is *close-to-optimal*.



# Performance Evaluation (Theoretical)

## Theorem

When the following three conditions hold:

(i)  $D(p_{ij}) = \max\{1 - \psi p_{ij}, 0\}$  for  $\psi > 0$  (linear demand function),

(ii)  $c^{\text{empty}} \rightarrow \infty$ ,

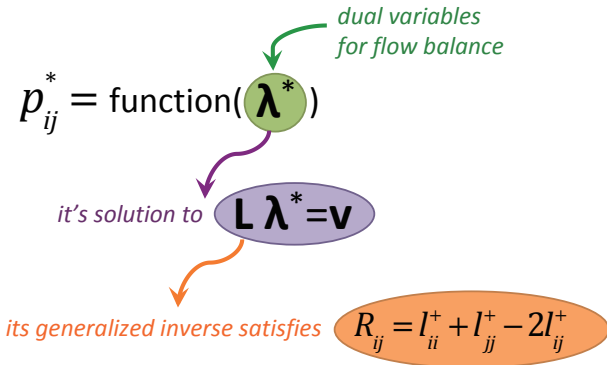
(iii) The auxiliary constraint  $p_{ij} \leq \frac{1}{\psi}$  is not binding,

the provider can **achieve the maximum profit** by choosing  $p_{ij}^{\psi}$ ,

$q_{ij}^{\text{full}} = \theta_{ij} D(p_{ij}^{\psi})$ , and  $q_{ij}^{\text{empty}} = 0$  for all  $i, j$ .

# Performance Evaluation (Theoretical)

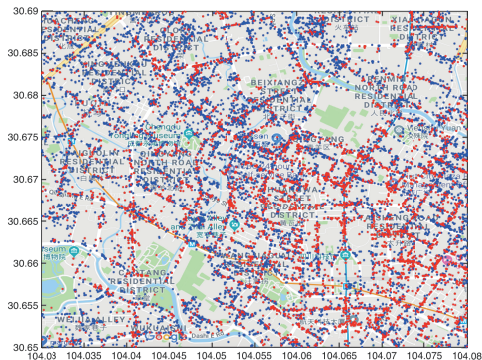
**Key Idea of Proof:** When conditions are satisfied,



# Performance Evaluation (Experimental)

We consider non-linear  $D(p_{ij})$  and finite  $c^{\text{empty}}$  in experiments.

- Real-world dataset (DiDi Chuxing GAIA Open Data Initiative)
  - Information of DiDi rides during November, 2016 in Chengdu. <sup>1</sup>



Pick-Up (Blue) and Drop-Off (Red) Dots During 7-9 am On Weekdays.

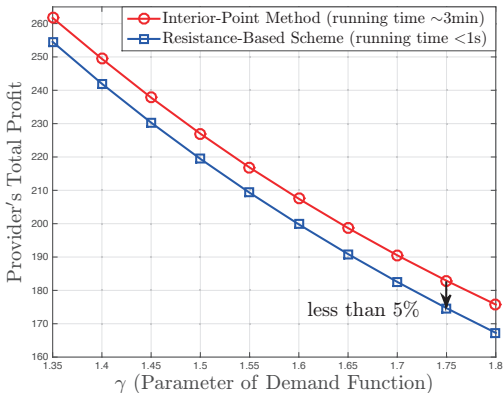
- We divide an area into 15 locations and derive  $\theta$  and  $\xi$ .

<sup>1</sup>DiDi Chuxing GAIA Open Data Initiative (<https://gaia.didichuxing.com>).

# Performance Evaluation (Experimental)

- Other experiment settings
  - $D(p_{ij}) = e^{-\gamma p_{ij}}$  (verified by real data in [Fang *et al.* 2017]).
  - $c^{\text{full}} = c^{\text{empty}} = 0.4$ .
  - $a_{ij}$  follows an exponential distribution with mean 0.15.
- Evaluated schemes
  - **Our resistance-based scheme** (complexity: polynomial in number of locations):
    - Choose  $p_{ij}^{\phi}$  for all  $i, j$  (where  $\phi = \frac{\gamma}{2}$ );
    - Choose  $q_{ij}^{\text{full}} = \theta_{ij} D(p_{ij}^{\phi})$  for all  $i, j$ ;
    - Choose  $q_{ij}^{\text{empty}}$  by solving an LP problem.
  - **Interior-point method**

# Performance Evaluation (Experimental)



Our resistance-based scheme achieves at least 95% of the profit achieved by the interior-point method.

# Conclusion

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  - Use **effective resistances** (capturing network topology's influence) to design prices.
  - Provide a simple and effective approach to measure location-based advertising's impact on network pricing.
- Other results in our work
  - Investigate the advertising's impact on users' payoffs.
  - Study the provider's optimal advertiser selection strategy.
- Future directions
  - Consider driver-side design in ride-sharing systems.
  - Consider time-variant traffic demand.

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**THANK YOU**