Cooperative Wi-Fi Deployment: A One-to-Many Bargaining Framework

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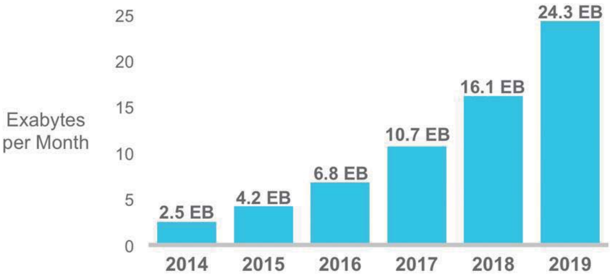
Background

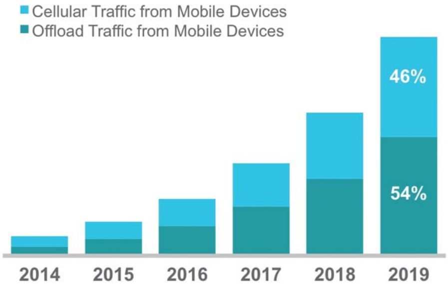


Global Mobile Data Traffic Growth till 2019 (©Cisco)

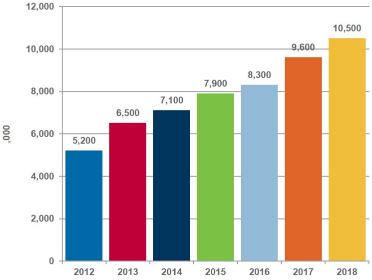
- Nearly 10-fold increase between 2014 and 2019

Background

- Need more Wi-Fi to offload cellular traffic
  - More than half of total traffic will be offloaded (54% by 2019)
  - The Wi-Fi deployment rate is increasing (10.5 million by 2018)



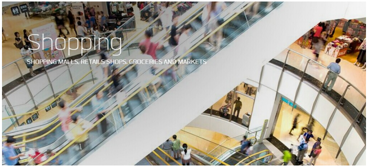
Percentage of Offloaded Traffic (©Cisco)



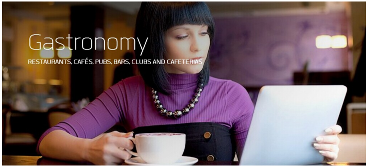
New Carrier-Grade Wi-Fi Per Year (©WBA)

Cooperative Wi-Fi Deployment

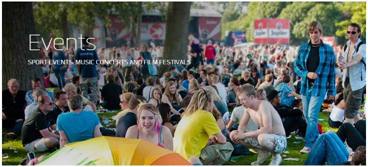
- MNOs cooperate with VOs to deploy public Wi-Fi
  - Venue owner: owner of public places



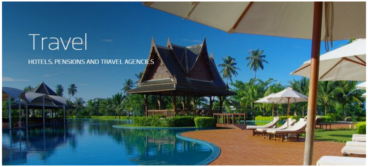
Shopping malls



Cafes



Stadiums



Hotels

Cooperative Wi-Fi Deployment

- MNOs cooperate with VOs to deploy public Wi-Fi
  - Example: AT&T (MNO) and Starbucks (VO)

AT&T provides Wi-Fi for Starbucks from 2008 to 2014

Problem Description

- Economic interactions between a monopoly MNO and multiple VOs:
  - Q1: *Which VOs should the MNO cooperate with?*
  - Q2: *How much should the MNO pay to these VOs?*
  - Q3: *What negotiation sequence can maximize the MNO's payoff?*

Model

- Basic Settings:
  - A set $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$ of VOs, describe VO n by (X_n, R_n, C_n);
  - $X_n \geq 0$: expected traffic offloaded by the Wi-Fi at venue n;
  - $R_n \geq 0$: extra revenue Wi-Fi creates for VO n;
  - $C_n \geq 0$: cost for the MNO to deploy Wi-Fi at venue n.
- Negotiation Results
  - $b_n \in \{0, 1\}$: whether the MNO cooperates with VO n;
  - $p_n \in \mathbb{R}$: the MNO's payment to VO n (could be negative)
  - For all $n \in \mathcal{N}$, define

$$\mathbf{b}_n \triangleq (b_1, b_2, \ldots, b_n),$$
$$\mathbf{p}_n \triangleq (p_1, p_2, \ldots, p_n).$$

Model

- Notations:
  - Negotiation variables (b_n, p_n), VO attributes (X_n, R_n, C_n)
- MNO's payoff:

$$U(\mathbf{b}_N, \mathbf{p}_N) = f\left(\sum_{n=1}^{N} b_n X_n\right) - \sum_{n=1}^{N} b_n C_n - \sum_{n=1}^{N} p_n$$

  - $f(\cdot)$ is increasing and strictly concave with $f(0) = 0$
- VO n's payoff:

$$V_n(b_n, p_n) = b_n R_n + p_n$$

Model

- Social welfare:

$$\begin{aligned}\Psi\left(\mathbf{b}_N\right) &= U\left(\mathbf{b}_N, \mathbf{p}_N\right) + \sum_{n=1}^{N} V_n\left(b_n, p_n\right) \\ &= f\left(\sum_{n=1}^{N} b_n X_n\right) + \sum_{n=1}^{N} b_n\left(R_n - C_n\right) \\ &= f\left(\sum_{n=1}^{N} b_n X_n\right) + \sum_{n=1}^{N} b_n Q_n\end{aligned}$$

  - Social welfare only depends on $\mathbf{b_N}$
  - $Q_n \triangleq R_n - C_n$ captures the factors excluding data offloading. In later analysis, describe VO n by (X_n, Q_n), instead of (X_n, R_n, C_n)

One-to-One Nash Bargaining

- Assume $|\mathcal{N}| = 1$, the problem degenerates to one-to-one bargaining
- NBS (Nash Bargaining Solution) solves:

$$
\begin{aligned}
\max \quad & (U(b_1, p_1) - U(0,0)) \cdot (V_1(b_1, p_1) - V_1(0,0)) \\
\text{s.t.} \quad & U(b_1, p_1) - U(0,0) \geq 0, V_1(b_1, p_1) - V_1(0,0) \geq 0, \\
\text{var.} \quad & b_1 \in \{0, 1\}, p_1 \in \mathbb{R}
\end{aligned}
$$

  - Disagreement points: $U(0,0) = V_1(0,0) = 0$
  - NBS maximizes the product of the players' payoff gains upon their disagreement points. With a higher disagreement point, the MNO (or VO) can obtain a larger payoff under the NBS.

One-to-One Nash Bargaining

- To simplify description, show the NBS in the form of (b_1^*, π_1^*), instead of (b_1^*, p_1^*): (π_1: VO 1's payoff; p_1: payment from MNO to VO 1)

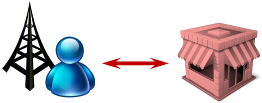
$$(b_1^*, \pi_1^*) = \begin{cases} \left(1, \frac{1}{2}\Psi(1)\right) & \text{if } \Psi(1) \geq \Psi(0) = 0, \\ (0,0) & \text{otherwise,} \end{cases}$$

  - If cooperation increases social welfare, the Wi-Fi will be deployed and they will equally share the generated revenue
  - Recall $\Psi(1) = f(X_1) + Q_1$, hence the NBS depends on X_1 and Q_1. That's to say, we only need to know Q_1, instead of (R_1, C_1)

Example 1: One-to-One Bargaining

Example 1

$f(x)=x^{0.5}$

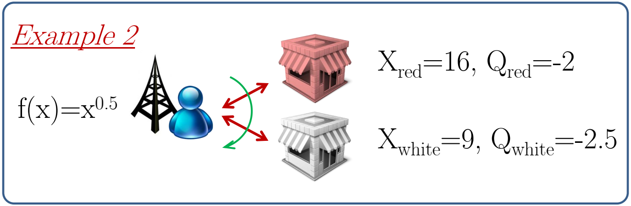


$X_{red}=16$, $Q_{red}=-2$

NBS: $b_{red}=1$, $\pi_{red}=1$

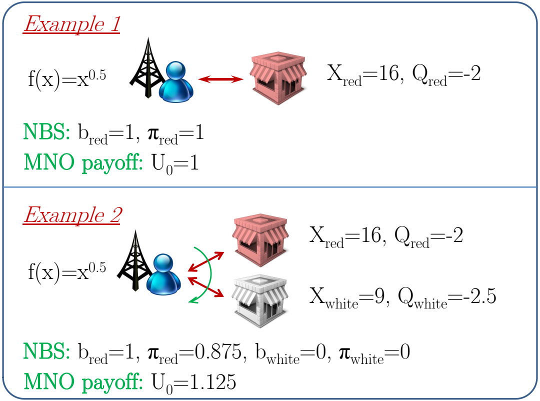
MNO payoff: $U_0=1$

Example 2: One-to-Many Sequential Bargaining



- Bargain with VOs sequentially: VO red→VO white
- Backward induction:
  - Step 2: assuming the MNO reaches (b_1, π_1) in Step 1, we study the one-to-one bargaining between the MNO and VO white;
  - Step 1: Based on VO white's response in step 2, we study the one-to-one bargaining between the MNO and VO red.

Example 2: One-to-Many Sequential Bargaining



- The existence of VO white allows the MNO to extract more revenue from the cooperation with VO red (think it as a backup plan)
- Different steps of bargaining generate externalities to each other, this is due to the concavity of the offloading benefit f

One-to-Many Bargaining I: Sequential Bargaining with Exogenous Sequence

Sequential Bargaining with Exogenous Sequence

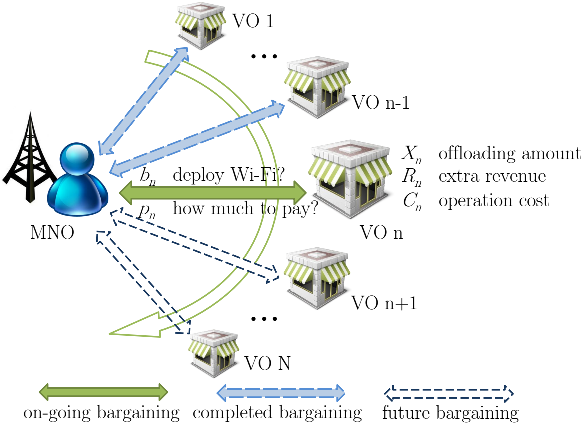


Figure: Illustration of Sequential Bargaining

Step N and Step N-1

NBS for step N

$$\left(b_N^*, \pi_N^*\right) = \begin{cases} \left(1, \frac{1}{2}\Delta_N\left(\mathbf{b}_{N-1}\right)\right) & \text{if } \Delta_N\left(\mathbf{b}_{N-1}\right) \geq 0, \\ (0,0) & \text{otherwise,} \end{cases}$$

Define $\Delta_N\left(\mathbf{b}_{N-1}\right) = \Psi\left(\left(\mathbf{b}_{N-1}, 1\right)\right) - \Psi\left(\left(\mathbf{b}_{N-1}, 0\right)\right)$

NBS for step N-1

$$\left(b_{N-1}^*, \pi_{N-1}^*\right) = \begin{cases} \left(1, \frac{1}{2}\Delta_{N-1}\left(\mathbf{b}_{N-2}\right)\right) & \text{if } \Delta_{N-1}\left(\mathbf{b}_{N-2}\right) \geq 0, \\ (0,0) & \text{otherwise,} \end{cases}$$

where we define

$$\begin{aligned} \Delta_{N-1}(\mathbf{b}_{N-2}) = {} & \Psi\left(\left(\mathbf{b}_{N-2}, 1, b_N^*\left(\left(\mathbf{b}_{N-2}, 1\right)\right)\right)\right) - \pi_N^*\left(\left(\mathbf{b}_{N-2}, 1\right)\right) \\ & - \Psi\left(\left(\mathbf{b}_{N-2}, 0, b_N^*\left(\left(\mathbf{b}_{N-2}, 0\right)\right)\right)\right) + \pi_N^*\left(\left(\mathbf{b}_{N-2}, 0\right)\right). \end{aligned}$$

Step k

NBS for step k

$$(b_k^*, \pi_k^*) = \begin{cases} \left(1, \frac{1}{2}\Delta_k\left(\mathbf{b}_{k-1}\right)\right) & \text{if } \Delta_k\left(\mathbf{b}_{k-1}\right) \geq 0, \\ (0,0) & \text{otherwise,} \end{cases}$$

where we define

$$\begin{aligned} \Delta_k\left(\mathbf{b}_{k-1}\right) = & \Psi\left(\left(\mathbf{b}_{k-1}, 1, b_{k+1}^*\left(\left(\mathbf{b}_{k-1}, 1\right)\right), \ldots, b_N^*\left(\left(\mathbf{b}_{k-1}, 1, , \ldots\right)\right)\right)\right) \\ & -\pi_{k+1}^*\left(\left(\mathbf{b}_{k-1}, 1\right)\right) - \ldots - \pi_N^*\left(\left(\mathbf{b}_{k-1}, 1, , \ldots\right)\right) \\ & -\Psi\left(\left(\mathbf{b}_{k-1}, 0, b_{k+1}^*\left(\left(\mathbf{b}_{k-1}, 0\right)\right), \ldots, b_N^*\left(\left(\mathbf{b}_{k-1}, 0, , \ldots\right)\right)\right)\right) \\ & +\pi_{k+1}^*\left(\left(\mathbf{b}_{k-1}, 0\right)\right) + \ldots + \pi_N^*\left(\left(\mathbf{b}_{k-1}, 0, \ldots\right)\right). \end{aligned}$$

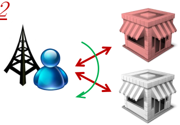
- Remark: The MNO's payoff under a particular bargaining sequence is fixed, and can be computed by a recursive algorithm (omitted)

One-to-Many Bargaining II: Sequential Bargaining with Endogenous Sequence

Influence of Bargaining Sequence

Example 2

$f(x)=x^{0.5}$



$X_{red}=16$, $Q_{red}=-2$

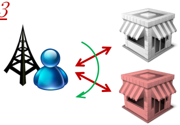
$X_{white}=9$, $Q_{white}=-2.5$

NBS: $b_{red}=1$, $\pi_{red}=0.875$, $b_{white}=0$, $\pi_{white}=0$

MNO payoff: $U_0=1.125$

Example 3

$f(x)=x^{0.5}$



$X_{white}=9$, $Q_{white}=-2.5$

$X_{red}=16$, $Q_{red}=-2$

NBS: $b_{white}=0$, $\pi_{white}=0$, $b_{red}=1$, $\pi_{red}=1$

MNO payoff: $U_0=1$

- Bargaining sequence affects cooperation outcomes, money transfer, social welfare, and the MNO's payoff

Optimal Sequencing Problem

- Question: Which bargaining sequence maximizes the MNO's payoff?
  - Cannot obtain the closed-form solution of the MNO's payoff
  - Checking all $|\mathcal{N}|!$ possibilities is time-consuming
- Key idea: prove structural properties related to VOs' types

Categorization of VOs and Structural Properties

Definition 1: VO Type

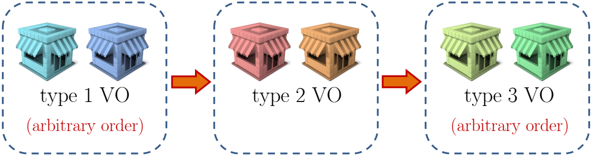
- VO $n \in \mathcal{N}$ belongs to:
  - *type* 1, if $Q_n \geq 0$;
  - *type* 2, if $Q_n < 0$ and $f(X_n) + Q_n \geq 0$;
  - *type* 3, if $Q_n < 0$ and $f(X_n) + Q_n < 0$.

- Observation:
  - Type 1 VO's cooperation with the MNO does not decrease the social welfare, *i.e.*, $\Psi(1, \mathbf{b}_{-n}) \geq \Psi(0, \mathbf{b}_{-n})$;
  - Type 2 VO's cooperation with the MNO may or may not decrease the social welfare, which depends on other VOs' attributes and positions;
  - Type 3 VO's cooperation with the MNO decreases the social welfare, *i.e.*, $\Psi(1, \mathbf{b}_{-n}) < \Psi(0, \mathbf{b}_{-n})$.

Categorization of VOs and Structural Properties

Theorem 1

- There exits a group of optimal bargaining sequences satisfying the following two conditions:
  (1) VO $l_1, l_2, \ldots, l_{N_1}$ are of type 1;
  (2) VO $l_{N_1+N_2+1}, l_{N_1+N_2+2}, \ldots, l_N$ are of type 3.
  For any optimal sequence that belongs to this group, if the MNO interchanges the bargaining positions of any two type 1 VOs (or two type 3 VOs), the MNO's payoff will not change.



Reduce Complexity from $|\mathcal{N}|!$ to $|\mathcal{N}_2|!$

- For example, there are 7 VOs, where $\{1,2\}$, $\{3,4,5\}$, $\{6,7\}$ are type 1, 2, 3, respectively.
  - By exhaust search, we need to check $7! = 5040$ possibilities;
  - By Theorem 1, we only need to check $3! = 6$ possibilities.

Special Case I: All Are Type 1

Theorem 2

- If all VOs are of type 1, the MNO's payoff is independent of the bargaining sequence **I** and is given as:

$$U_0 = \frac{1}{2^N} \sum_{\mathbf{b}_N \in \mathcal{B}} \Psi\left(\mathbf{b}_N\right),$$

where $\mathcal{B} \triangleq \{(b_1, b_2, \ldots, b_N) : b_n \in \{0, 1\}, \forall n \in \mathcal{N}\}$.

- Remark: Can write down the close-form solution of the MNO's payoff

Special Case II: All Are Sortable

Definition 2

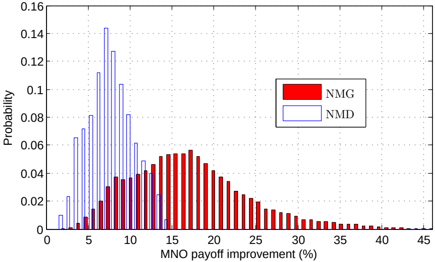
- A set $\mathcal{N}$ of VOs is said to be sortable if and only if for any $i, j \in \mathcal{N}$, we have $(X_i - X_j)(Q_i - Q_j) \geq 0$.

Theorem 3

- If all VOs are sortable, we can construct a sequence $\mathbf{I}$ such that $X_{I_n} \geq X_{I_{n+1}}$, $Q_{I_n} \geq Q_{I_{n+1}}$, $\forall n \in \{1, 2, \ldots, N-1\}$. Furthermore:
  (1) $\mathbf{I}$ is the optimal bargaining sequence;
  (2) Under $\mathbf{I}$, the MNO will and only will cooperate with VO $I_1, I_2, \ldots, I_k$, where $k \in \{0\} \cup \mathcal{N}$ is uniquely determined by two inequalities (omitted here)

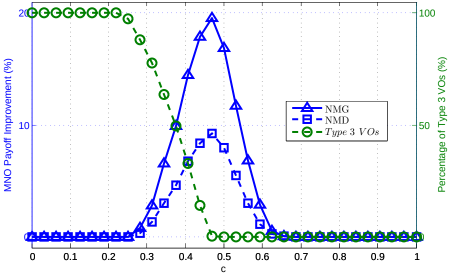
- Remark: Can quickly provide the optimal sequence without any searching

Simulation 1: Advantage of Optimal Sequencing



- Settings: $f(x) = x^{1/2}$, $|\mathcal{N}| = 5$
- Compared with the worst sequence, the optimal sequence improves the MNO's payoff by 17% on average and by 46% at most;
- Compared with the random sequence, the optimal sequence improves the MNO's payoff by 8% on average and by 15% at most.

Simulation 2: Influence of Offloading Benefit



- Settings: $f(X) = X^c$, $|\mathcal{N}| = 4$
- Optimal sequencing's advantage is not obvious for small and large c
  - Small c: offloading benefit is small, hence most VOs are type 3, and the MNO does not cooperate with these VOs
  - Large c: function $f(\cdot)$'s concavity is small and the externalities among different steps of bargaining are weak

Conclusion

- Study cooperative public Wi-Fi deployment
- Consider one-to-many Nash bargaining
  - Exogenous bargaining sequence: analyze the MNO's payoff under a given bargaining sequence, with the consideration of externalities among VOs
  - Endogenous bargaining sequence: obtain the optimal bargaining sequence by leveraging the structural property

THANK YOU